Exercise 1

Find the principal argument $\operatorname{Arg} z$ when

$$(a)\ z = \frac{i}{-2-2i}; \qquad (b)\ z = (\sqrt{3}-i)^6.$$

$$Ans.\ (a)\ -3\pi/4; \quad (b)\ \pi.$$

Solution

Part (a)

$$\begin{aligned}
\arg z &= \arg\left(\frac{i}{-2-2i}\right) \\
&= \arg(i) - \arg(-2-2i) \\
&= \left(\frac{\pi}{2}\right) - \left(\tan^{-1}\frac{-2}{-2} + \pi\right) + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots \\
&= \frac{\pi}{2} - \left(\frac{\pi}{4} + \pi\right) + 2n\pi \\
&= -\frac{3\pi}{4} + 2n\pi
\end{aligned}$$

Since the principal argument $\operatorname{Arg} z$ is required to be between $-\pi$ and π ($-\pi < \operatorname{Arg} z \leq \pi$), we choose $n = 0$.

$$\operatorname{Arg} z = -\frac{3\pi}{4}$$

Part (b)

Switch to polar form first to deal with the exponent.

$$\begin{aligned}
\arg z &= \arg\left(\sqrt{3} - i\right)^6 \\
&= \arg\left[\sqrt{(\sqrt{3})^2 + (-1)^2}\exp\left(i\tan^{-1}\frac{-1}{\sqrt{3}}\right)\right]^6 \\
&= \arg\left[\sqrt{4}\exp\left(-i\frac{\pi}{6}\right)\right]^6 \\
&= \arg\left(2e^{-i\pi/6}\right)^6 \\
&= \arg\left(64e^{-i\pi}\right) \\
&= -\pi + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots
\end{aligned}$$

Since the principal argument $\operatorname{Arg} z$ is required to be between $-\pi$ and π ($-\pi < \operatorname{Arg} z \leq \pi$), we choose $n = 1$.

$$\operatorname{Arg} z = \pi$$